FREE TUTORIAL ON MATHEMATICS

TOPIC: LOGARITHMS

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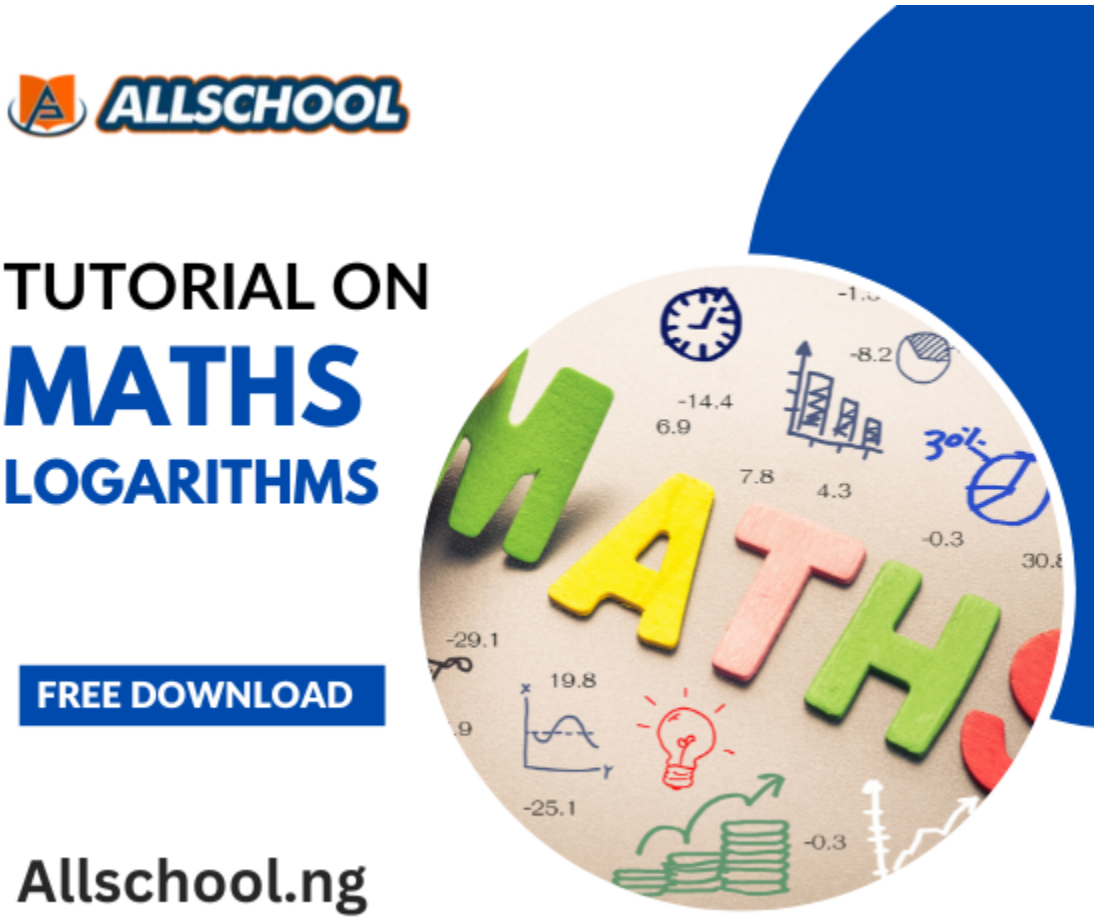
LOGARITHMS



hematical functions that represent the exponent to which a specific base must be raised to obtain a given number.

They are widely used in various fields, including algebra, calculus, and computer science.

The logarithm of a number x to the base b is denoted as $\log_b(x)$ or simply $\log(x)$ when the base is 10.

Properties of Logarithms:

1. **Product Rule**: $\log_b(xy) = \log_b(x) + \log_b(y)$
2. **Quotient Rule**: $\log_b(x/y) = \log_b(x) - \log_b(y)$

3. **Power Rule**: $\log_b(x^n)=n \cdot \log_b(x)$
4. **Change of Base Formula**: $\log_b(x)=\log_c(x) / \log_c(b)$

Note: (·) in mathematics means multiply

Examples:

Example 1: Solve for x in the equation $2^x=8$.

Short solution:

$x = \log_2(8)$
$= \log_2(2^3)$
$= 3$

Long Explanation:

1. Recognize that $2^x=8$ can be rewritten using logarithms as $\log_2(8)=x$.
2. Apply the definition of a logarithm to write the equation in exponential form: So $2^x = 8$ is equivalent to $x = \log_2(8)$.
3. Evaluate the logarithm: $\log_2(8)$ is asking, "*To what power must 2 be raised to get 8*?"
4. The answer is 3 because 2^3 (i.e. 2 x 2 x 2) = 8.
5. Therefore, the answer is $x = 3$.

Example 2: Simplify the expression $\log_5(25) + \log_5(⅕)$.

Short explanation:

$\log_5(25) + \log_5(⅕) = \log_5(25 \cdot ⅕)$
$=\log_5(5)$
$=1$

HOT TIP: Whenever you see $\log_{\mathbf{a}}(\mathbf{a})$, for example, $\log_{\mathbf{5}}(\mathbf{5})$, $\log_{\mathbf{2}}(\mathbf{2})$, or $\log_{\mathbf{7}}(\mathbf{7})$ the answer will always be **1**.

Long Explanation:

1. Use the product rule of logarithms, which states that $\log_b(xy) = \log_b(x) + \log_b(y)$.
2. Apply the **product rule** to combine the two logarithms: $\log_5(25) + \log_5(⅕) = \log_5(25 \cdot ⅕)$
3. Simplify the expression inside the logarithm: $25 \cdot ⅕ = 5$.
4. Therefore, the simplified expression is $\log_5(5)$.
5. Evaluate the logarithm: $\log_5(25)$ is asking, "To what power must 5 be raised to get 5?" The answer is **1**.
6. So, the final result is 1.

Exercises:

Make sure you try solving these questions yourself. It'll help you understand the topic very well.

1. Solve for x: $3^x = 27$.
2. Simplify: $\log_2(4) + \log_2(8)$
3. If $\log_a(b) = 2$ and $\log_a(c) = 3$, find $\log_a(bc)$.
4. Solve for x: $e^x = 20$.
5. Simplify: $2\log_3(5) - \log_3(125)$

SOLUTIONS TO EXERCISES

> *Hey, don't cheat yourself. Make sure you attempt the exercise before checking the solution.*

Exercise 1: Solve for x: $3^x = 27$.

Short solution: $x = \log_3(27) = \log_3(3^3) =$ **3**.

Long Solution:

1. Apply the definition of logarithms to rewrite the equation as $x = \log_3(27)$.
2. Evaluate the logarithm: $\log_3(27)$ is asking, "*To what power must 3 be raised to get 27?*" The answer is 3 (because 3 x 3 x 3 is 27).
3. Therefore, the solution is $x = 3$.

Exercise 2: Simplify: $\log_2(4) + \log_2(8)$

Short Solution: $\log_2(4) + \log_2(8) = \log_2(4 \cdot 8) =$ **$\log_2(32)$**

Long Solution:

1. Use the product rule to combine the two logarithms: $\log_2(4) + \log_2(8) = \log_2(4 \cdot 8)$
2. Simplify the expression **inside** the logarithm: $4 \cdot 8 = 32$.
3. Therefore, the simplified expression is **$\log_2(32)$**.

Exercise 3: If $\log_a(b) = 2$ and $\log_a(c) = 3$, find $\log_a(bc)$.

Short solution: $\log_a(bc) = \log_a(b) + \log_a(c) = 2+3 =$ **5**

Long Explanation:

1. Use the product rule of logarithms: $\log_a(bc) = \log_a(b) + \log_a(c)$
2. Substitute the given values: $\log_a(bc) = 2+3 = 5$.
3. Therefore, $\log_a(bc) =$ **5**.

Exercise 4: Solve for x: $e^x = 20$.

Short solution: $x = \ln(20) =$ **2.996**

Long Explanation:

$e^x = 20$
Whenever you see this kind of question, just apply the natural logarithm (denoted as **ln**) to both sides to solve for x

So you will have $\mathbf{ln}(e)^x = \mathbf{ln}(20)$

Natural Logarithm always cancels exponential.

So we'll just have $x =$ **ln**(20)

If you punch **ln**(20) in your calculator, you'll get **2.996**.

Exercise 5: Simplify: $2\log_3(5) - \log_3(125)$

Short solution: $2\log_3(5) - \log_3(125) = \log_3(5^2 / 5^3) =$ **$\log_3(\frac{1}{5})$**

Detailed explanation:

1. Use the power rule of logarithms: $2\log_3(5)$ can be written as $\log_3(5^2)$.
2. Substitute this back into the expression: $\log_3(5^2) - \log_3(125)$
3. Simplify: $5^2 = 25$, so the expression becomes $\log_3(25) - \log_3(125)$
4. Apply the quotient rule: $\log_3(25) - \log_3(125) = \log_3(25 / 125)$.

5. Simplify the fraction inside the logarithm: 25 / 125 = 1/5.
6. Therefore, the simplified expression is $\log_3(⅕)$.